

Cosmic string network evolution in arbitrary Friedmann–Lemaître models

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Abstract

We use the modified “one-scale” model by Martins & Shellard to investigate the evolution of a GUT long cosmic string network in general Friedmann–Lemaître models. Four representative cosmological models are used to show that in general there is no scaling solution. The implications for structure formation in these models are briefly discussed.

1 Introduction

Cosmic strings might be responsible for structure formation in the universe. The theory predicts three mechanism for structure formation: wake formation by fast moving long strings, accretion of matter by cosmic string loops and filamentary accretion by slow moving long strings. Which of these mechanism is important depends on the evolution of the cosmic string network (for a review see [1],[2]).

Up to now, the cosmic string scenario of structure formation has been investigated only in the Einstein–de Sitter model, in which the cosmic string network reaches a scaling solution, i.e. the typical length scale of the network scales with the Hubble radius. Investigations in open universes assumed such a scaling solution a priori [3]. Recent work indicates that, if the network reaches scaling, the angular power spectrum C_l and the COBE normalised matter power spectrum doesn’t fit the observation [4].

However, in recent years it became more and more obvious, that the Einstein–de Sitter model is in conflict with some astronomical observations, namely the age of the oldest stars [5], the baryonic content of X–Ray clusters [6] and the line distribution of hydrogen absorbers in the Lyman– α –forest [7]. Therefore, it is necessary to investigate the evolution of a cosmic string network in more general Friedmann–Lemaître models. The first quantitative discussion of the evolution of a cosmic string network in open models was given by Martins [8]. In this paper we extend his analysis and discuss the evolution of a GUT cosmic string network in more general cosmological Friedmann–Lemaître models. There are several types which are interesting in modern cosmology. The flat models (some of them could be produced in an inflationary epoch, but not in general) have $\Omega + \lambda = 1$, where $\Omega(t) = 8\pi G\rho(t)/(3H^2(t))$ is the density parameter and $\lambda(t) = \Lambda/(3H^2(t))$ is the normalized cosmological term (Λ is the cosmological constant). The open models with $\Omega < 1$ are favoured by the measurements of

cluster masses [9], but $\Lambda = 0$ was assumed for the resulting cosmological model. The third class of interesting cosmological models are closed with a loitering phase of slow expansion. The time and the duration of the phase of slow expansion depends on the present values of Ω and λ . Such a model is suggested by the Ly α -absorption lines distribution in quasar spectra [7]. They are considered by several authors in the context of structure formation [10].

Numerical simulations are the simplest (and most expensive) ways to study the cosmic string network evolution. Other possibilities are analytical models. The first model was introduced by Kibble, the “one-scale” model [11]. In this model, the fundamental quantity is a typical length scale $L(t)$, defined by

$$\rho_{\infty}(t) \equiv \mu/L^2(t), \quad (1)$$

where ρ_{∞} is the energy density in long strings and μ is the mass per unit length on the string (we set $c = 1$). With more detailed numerical studies it became obvious, that long strings are not straight but contain wiggles [12][13]. Therefore, Austin, Copeland and Kibble modified the “one-scale” model and introduced two new length scales in order to describe these wiggles [14]. Another model was introduced by Martins & Shellard [15], in which the RMS velocity of the strings are treated as a fundamental, independent quantity. We use this model, to study the evolution of a cosmic string network in four representative FL models. This velocity-dependent “one-scale” model (VDOSM) is briefly described in section 2. In section 3 we present our calculations. We discuss the results and implications on structure formation and anisotropies of the CMBR in section 4.

2 The velocity-dependent “one-scale” model

There are only two macroscopic quantities in the VDOSM. The first is the energy of a piece of string:

$$E = \mu a(\tau) \int \epsilon d\sigma, \quad (2)$$

where ϵ is the energy per length σ on the string. The other quantity is the RMS velocity of the (long) string

$$v_{\infty}^2 = \frac{\int \dot{\mathbf{x}}^2 \epsilon d\sigma}{\int \epsilon d\sigma}. \quad (3)$$

The typical length scale L in the network is defined in eq. (1). One has to include several phenomenological parameter, the first one is the “loop chooping efficiency” \tilde{c} , defined by

$$\left(\frac{d\rho_{\infty}}{dt} \right)_{\text{to loops}} = \tilde{c} v_{\infty} \frac{\rho_{\infty}}{L}. \quad (4)$$

Note that in the original “one-scale” model the velocity was absorbed in the definition of \tilde{c} . For our purposes the loop reconnection onto long strings is negligible, as indicated by numerical simulations[12][13]. The scaling properties depend **not** crucial on the parameter \tilde{c} .

Neglecting the effects of frictional forces, the equation for the evolution of the length scale L can be obtained from eqs. (2)–(4) and is given by:

$$\frac{dL}{dt} = HL(1 + v_{\infty}^2) + \frac{\tilde{c} v_{\infty}}{2}. \quad (5)$$

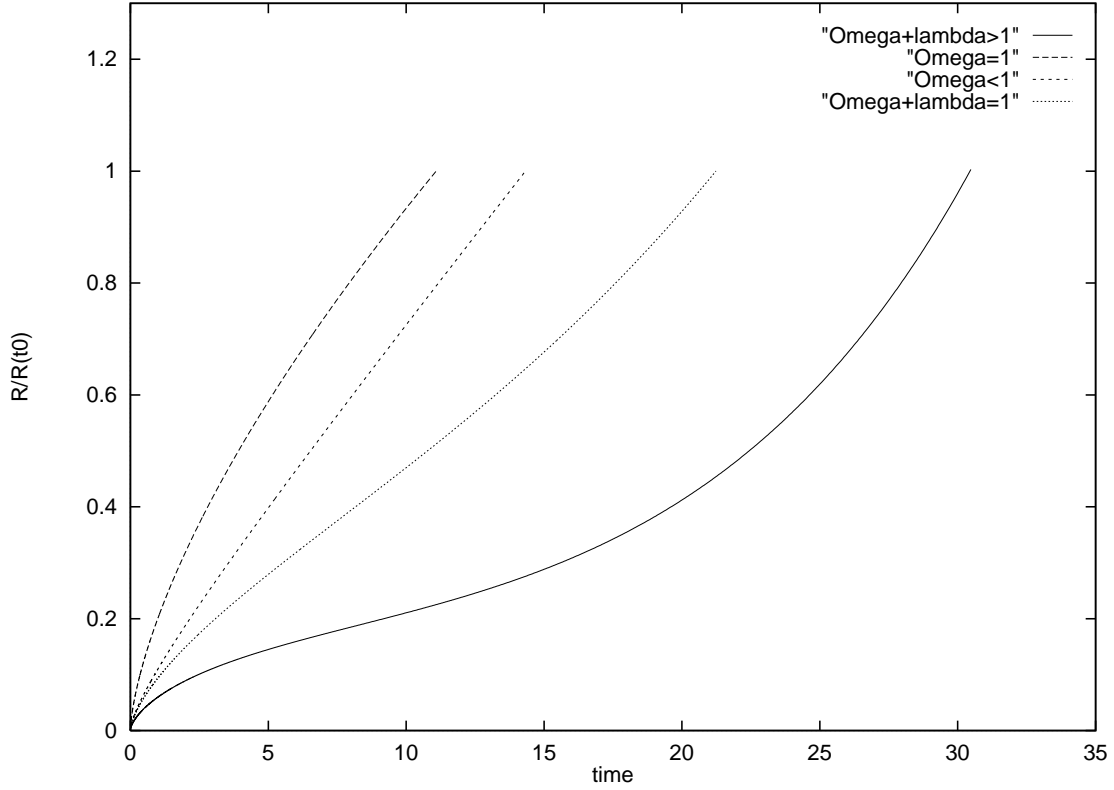


Figure 1: The scale-factor ($R(t)/R_0$) as a function of time (in units of 10^9 years) for the four representative models.

The evolution of the relevant length scale of string loops is described by

$$\frac{dl}{dt} = (1 - 2v_l^2)Hl - \Gamma' G_\mu v_l^6 \quad (6)$$

where $\Gamma' = 8 \times 65$. The second term describes the decay of the loops due to gravitational radiation. Finally, the evolution of the RMS velocity is given by

$$\frac{dv}{dt} = (1 - v^2) \left(\frac{k}{r} - 2Hv \right), \quad (7)$$

where k is another phenomenological parameter that is related to the small scale structure on the strings. An appropriate ansatz for it is

$$k = \begin{cases} 1, & 2Hr > \chi \\ \sqrt{2}Hr, & 2Hr < \chi \end{cases}$$

Here r is the curvature radius of the string, i.e. $r = L$ for long strings and $l = 2\pi r$ for loops. χ is a numerically determined coefficient of order unity, see the paper by Martins & Shellard [15] for a complete discussion of this point.

Model	K	Ω_0	λ_0	$H_0/(\text{km}/(\text{s} \cdot \text{Mpc}))$
1	+1	0.014	1.08	90
2	0	1.0	0.0	60
3	-1	0.1	0.0	60
4	0	0.1	0.9	60

Table 1: The four representative cosmological models.

3 Computations

The evolution of the scale factor R is described by the Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho_{\text{matter}} + \rho_{\text{radiation}} + \rho_{\infty}) + \frac{\Lambda}{3} - \frac{K}{R^2}. \quad (8)$$

Here $H = \dot{R}/R$ is the Hubble parameter and R is the scale factor. K represents the topology of the space and is zero for a flat universe, -1 for an open universe and $+1$ for a closed universe. Λ is the cosmological constant. We analyse four representative models, a open (hyperbolic) model, a flat model with a cosmological constant, a closed universe with a cosmological constant and the (flat) Einstein–de Sitter model. The behaviour of the scale factor for these models is plotted in Figure 1. These four models represent the interesting class of models in modern cosmology. The flat model with a cosmological constant could be introduced to retain the flatness while lowering Ω_0 . The open model was introduced in favour for a low Ω_0 with $\Lambda = 0$. The closed model was obtained from the Ly- α -forest [7], by assuming a constant comoving absorber density and represents the class of loitering models, in which the universe undergoes a epoch of slow expansion at a redshift about 5.

We solve the equations numerically with the standard Runge–Kutta method. Our results are presented in Figures 2 to 6. Our results for the Einstein–de Sitter model and for the open model are in agreement with the calculations by Martins [8].

One can see, that only in the Einstein–de Sitter model the network approaches a scaling regime. As pointed out by Martins, this is easy to understand: In a universe where the scale factor grows as $R \propto t^s$ ($s < 1$), one finds for in the linear regime

$$\left(\frac{L}{t}\right)^2 = \frac{k(k + \tilde{c})}{4s(1 - s)} \quad (9)$$

and

$$v^2 = \frac{k(1 - s)}{s(k + \tilde{c})}. \quad (10)$$

In the Einstein–de Sitter model s varies only from $s = 1/2$ to $s = 2/3$ (radiation to matter dominated). In the other models, however, there are several other epochs, namely curvature dominated epochs and vacuum dominated epochs. In the vacuum dominated epochs, the scale factor grows as $R \propto \exp(t)$, therefore s is a function of time in these epoch, i.e. there is no scaling solution.

We don't plot the ratio $\rho_{\infty}/\rho_{\text{loops}}$, where ρ_{loops} is the energy density in loops, because we arrive the same conclusions as Martins for the open model and the Einstein–de Sitter model.

In the other two models the strings will never dominate the energy density of the universe, first because in models with a cosmological constant L increases more rapidly than in the open model, and second the λ -Term approaches 1 for $t \rightarrow \infty$, a value which could never be reached by ρ_∞ or ρ_{loops} .

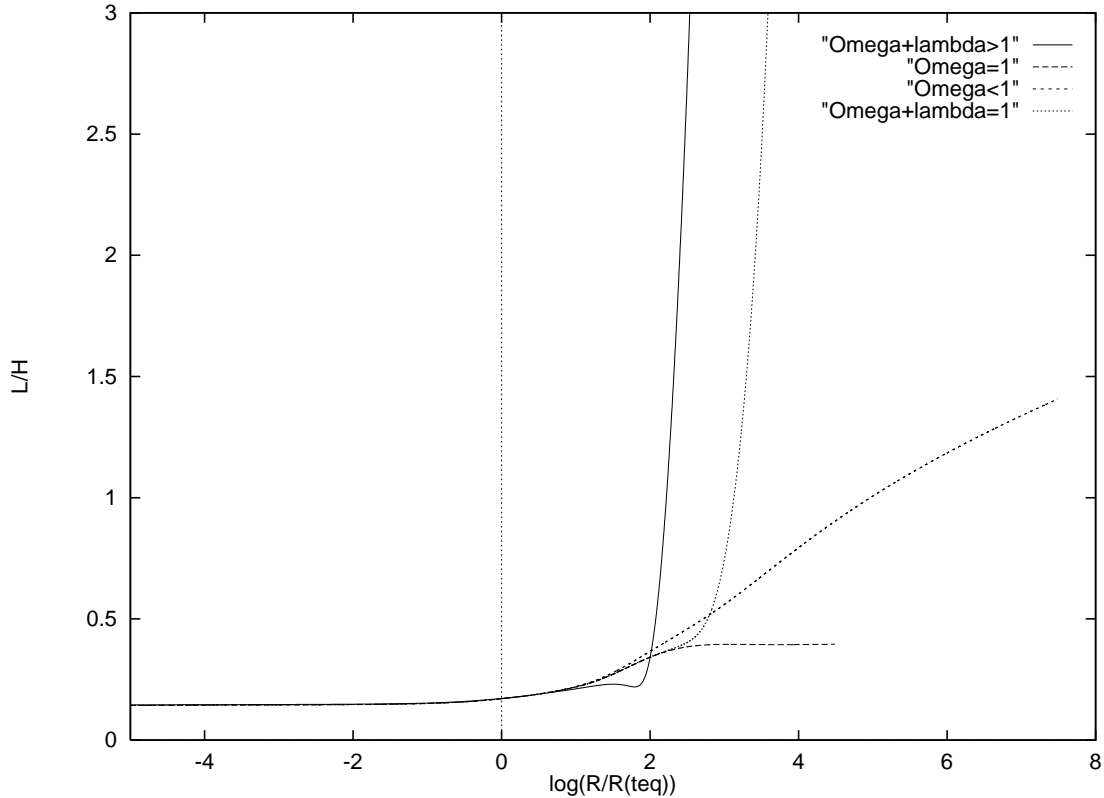


Figure 2: Ratio L/H as a function of $\log(R/R_{eq})$.

4 Discussion

The fact, that there is no scaling solution in the general case has important consequences on the structure formation theory with cosmic strings. In open universes one expects differences (compared to the Einstein-de Sitter model) only on large scales. This could have important consequences on the normalisation of the string mass per unit length μ from the COBE data. The same holds for the flat model with $\Omega_0 + \lambda_0 = 1$. We expect significant consequences in the closed model with $\Omega_0 + \lambda_0 > 1$. This is due to the fact, that the loop production rate is higher than in the other models (see Figure 4). Thus, loops could play an important role in structure formation in this model. One can also see, that the RMS velocity is high, suggesting, that the wiggly strings produce wakes rather than filaments. If our results can solve the problem of structure formation with cosmic strings [4], should be investigated in more detail.

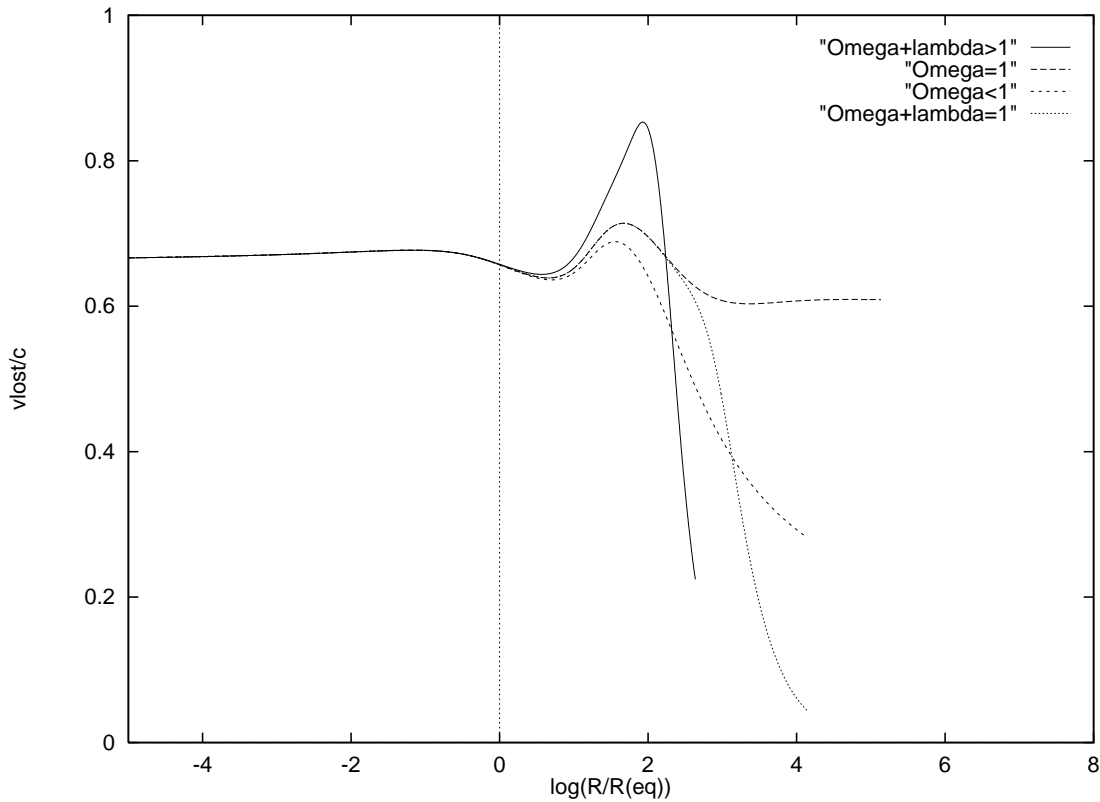


Figure 3: Long string RMS velocity in units of c as a function of $\log(R/R_{eq})$.

Our results have not only consequences on structure formation theory with cosmic strings. The prediction of the gravitational wave background and the spectrum of high-energy particles depends also on the network evolution.

Our work based on the “velocity-dependent” one-scale model by Martins & Shellard. If this model can describe all transition regimes (for example from matter to vacuum regimes) and if the ansatz for k is correct will be investigated in more detail in future publications.

Structure formation and the anisotropies in the CMBR due to long cosmic strings in these cosmological models are investigated our future work.

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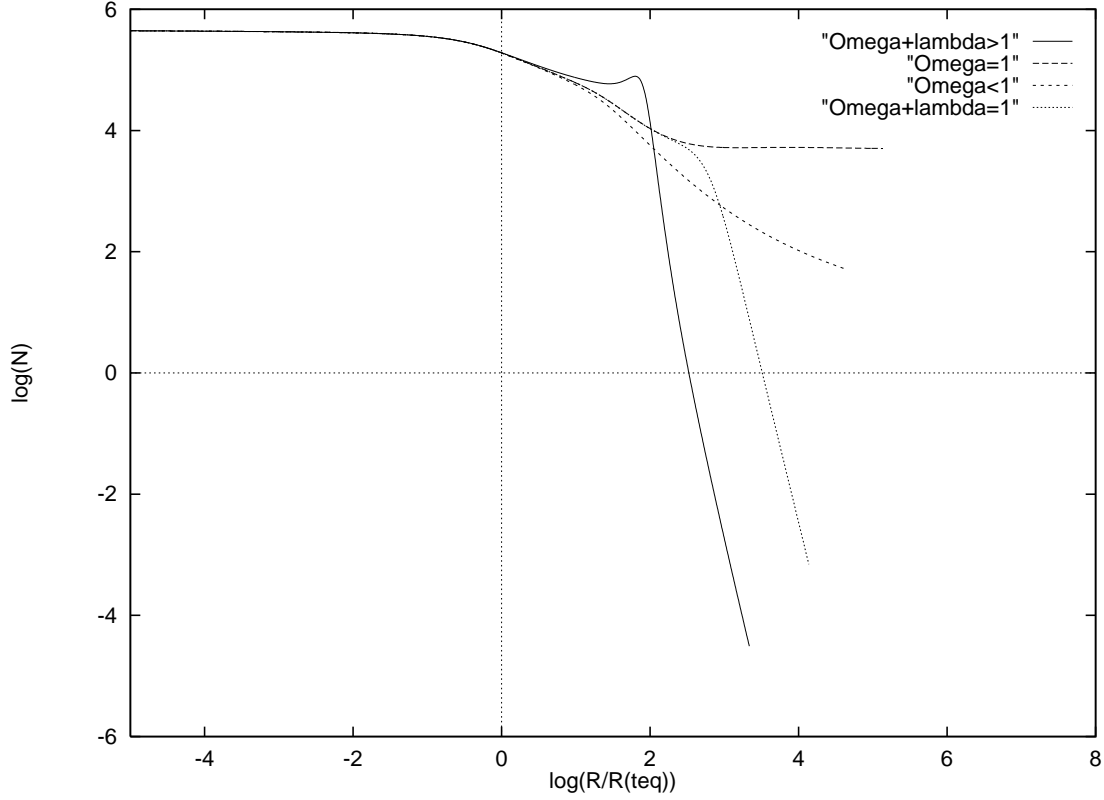


Figure 4: The logarithm of the number N of loops produced per Hubble volume and Hubble time as a function of $\log(R/R_{eq})$.

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